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MICROMECHANICAL FORMULAS FOR THE RELAXATION TENSOR OF LINEAR VISCOELASTIC COMPOSITES WITH TRANSVERSELY ISOTROPIC FIBERS

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Abstract—Explicit analytical expressions for the relaxation moduli in the Laplace domain of composites with viscoelastic matrix and transversely isotropic fibers are developed. The correspondence principle in viscoelasticity is applied and the problem in the Laplace domain is studied using the solution of the elastic problem having periodic microstructure. Formulas for the Laplace transform of the relaxation functions of the composite are obtained in terms of the properties of the matrix and the fibers. The inversion to the time domain of the relaxation and the creep functions of composites reinforced by transversely isotropic fibers is carried out numerically when a power law model is applied to represent the viscoelastic behavior of the matrix. Finally, comparisons with experimental results are presented.

1. INTRODUCTION

The creep response of polymer and metal matrix composites is one of the limiting design parameters for advanced composite structures expected to operate for long periods of time on a variety of applications (Barbero, 1994). The widespread use of carbon fibers provides the motivation for the development of a model capable of representing composite materials reinforced with transversely isotropic fibers.

Many micromechanical models have been developed to estimate the elastic properties of composite materials (Christensen, 1990; Mura, 1987). The macroscopic viscoelastic properties of fiber-reinforced materials were evaluated by the cylinder assemblage model proposed by Hashin (1965, 1966), where the correspondence principle (Christensen, 1979) was applied. Christensen (1969) proposed an approximate formula for the effective complex shear modulus in the case of materials with two viscoelastic phases by using the composite sphere model. The self-consistent method and a numerical inversion method were used by Laws and McLaughlin (1978) to obtain the response in the time domain. Yancey and Pindera (1990) estimated the creep response of unidirectional composites with linear viscoelastic matrices and transversely isotropic elastic fibers by applying the micromechanical model proposed by Aboudi (1991) to obtain the Laplace transform of the effective viscoelastic moduli. Then, they used Bellman's numerical method for the inversion to the time domain. Wang and Weng (1992) used the Eshelby–Mori–Tanaka method (Mori and Tanaka, 1973) in order to obtain the overall linear viscoelastic properties.

While several micromechanical models initially developed for the analysis of the elastic behavior of composites have been extended to the viscoelastic case, no model has been developed for linear viscoelastic solids with periodic microstructure, even though many results are available for the elastic case (Nemat-Nasser and Hori, 1993). Furthermore, no attempt has been made to develop explicit analytical expressions in the time domain.

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Interest in using the elastic solution for composites with periodic microstructure (Luciano and Barbero, 1994a) is motivated by the following reasons. First, the periodic solution is the best approximation available for advanced composites that have periodic microstructure, taking into account not only the periodicity of the microstructure but also the geometry of the inclusions. For composites without perfectly periodic microstructure, the periodic solution provides a bound on the estimate of the overall properties, while other methods (e.g. the self consistent method) provide a bound that represents the perfectly random distribution of the inclusions (Nemat-Nasser and Hori, 1993).

In the present paper, analytical expressions in the Laplace and time domain for the coefficients of the creep and relaxation tensors of composite materials with periodically distributed transversely isotropic fibers and linear viscoelastic matrix are proposed. The inversion in the time domain is carried out numerically because of the nonlinearity of the viscoelastic behavior of the matrix, which is represented by a power law. Comparisons with available experimental data obtained by Yancey and Pindera (1990) and other results that illustrate the capability of the model are presented.

2. VISCOELASTIC CONSTITUTIVE EQUATIONS

The constitutive equations of a linear viscoelastic isotropic material can be expressed in the time domain in the following way:

$$\sigma(t) = I^{(2)} \int_{-\infty}^{t} \lambda(t-\tau) tr\dot{\varepsilon}(\tau)\,\mathrm{d}\tau + 2\int_{-\infty}^{t} \mu(t-\tau)\dot{\varepsilon}(\tau)\,\mathrm{d}\tau, \tag{1}$$

where $\sigma(t)$ and $\varepsilon(t)$ are the stress and strain tensor, $\lambda(t)$ and $\mu(t)$ are the two stress-relaxation functions, the dot indicates the differentiation with respect to time and $I^{(2)}$ denotes the identity tensor of second order.

If the Laplace transform of a function $f(t)$ is denoted as:

$$\tilde{f}(s) = \int_{0}^{\infty} f(t)\exp(-st)\,\mathrm{d}t, \tag{2}$$

then eqn (1) can be expressed in the following way:

$$\tilde{\sigma}(s) = s\tilde{\lambda}(s) tr\tilde{\varepsilon}(s) I^{(2)} + 2s\tilde{\mu}(s)\tilde{\varepsilon}(s) = s\tilde{C}(s)\tilde{\varepsilon}(s). \tag{3}$$

The Poisson ratio in the transformed domain ν^{TD} is written in terms of $\tilde{\lambda}(s)$ and $\tilde{\mu}(s)$ as:

$$\nu^{\mathrm{TD}} = \tilde{\lambda}(s)/2(\tilde{\lambda}(s)+\tilde{\mu}(s)). \tag{4}$$

In the following, only the set of linear viscoelastic materials with Poisson ratio that remains constant in the course of the deformation [i.e. $\nu(t) = \nu = \nu^{\mathrm{TD}}$] will be analysed [see Aboudi (1991) and Wang and Weng (1992)].

3. PERIODIC EIGENSTRAIN IN THE LAPLACE DOMAIN

Consider an infinitely extended linearly viscoelastic solid with periodic microstructure characterized by a unit cell D (Fig. 1). Let each cell D be a parallelepiped with dimensions a_j in the direction of the coordinate axes x_j, where $j = 1, 2, 3$ and let V be its volume. The unit cell is divided into two parts: the fiber Ω and the matrix D-Ω and let V_f be the volume fraction of the fibers ($V_f = V_\Omega/V$).

In the following a composite with viscoelastic isotropic matrix and elastic transversely isotropic inclusions will be considered. The viscoelastic behavior of the matrix will be

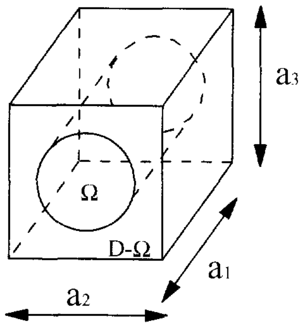

Fig. 1. Geometry of the unit cell D.

represented by eqn (1) or (3), while the constitutive equation for the transversely isotropic fibers is written as:

$$\tilde{\sigma}(s,x) = s\tilde{C}'(s)\tilde{\varepsilon}(s,x) = C'\tilde{\varepsilon}(s,x) \text{ in } \Omega. \tag{5}$$

In order to compute the viscoelastic properties of the composite, the fibers, periodically distributed in the body, can be simulated in the Laplace domain by an homogenization eigenstrain $\tilde{\varepsilon}^*(s,x)$, periodic in x_j. In this way, the analysis of the viscoelastic behavior of a composite material with periodic microstructure is reduced to the viscoelastostatic problem of a solid subject to a periodic eigenstrain. The expression of the Laplace transform of the periodic strain $\tilde{\varepsilon}(s,x)$ inside Ω in terms of $\tilde{\varepsilon}^*(s,x)$ can be derived (Luciano and Barbero, 1994b) from the elastic expression (Luciano and Barbero, 1994a). However, in order to obtain the relaxation moduli of the composite it is not necessary to have the exact expression of $\tilde{\varepsilon}(s,x)$, but only its volume average in Ω, $\overline{\tilde{\varepsilon}(s)}$. In particular, the volume average of the strain in the inclusion $\overline{\tilde{\varepsilon}(s)}$ can be written in terms of the volume average of the eigenstrain $\overline{\tilde{\varepsilon}^*(s)}$ in the following way:

$$\overline{\tilde{\varepsilon}(s)} = P(s) : \tilde{C}(s) : \overline{\tilde{\varepsilon}^*(s)} = S(s) : \overline{\tilde{\varepsilon}^*(s)}$$

where $S(s)$ is the Eshelby tensor in the Laplace domain for solids with periodic microstructure and depends only on the periodicity of the unit cell (a_j), the geometry of the inclusions and the viscoelastic properties of the matrix.

If the expression of $S(s)$ is known, the equivalent eigenstrain method can be applied in order to obtain the exact volume-average homogenization eigenstrain which simulates the presence of the periodic fibers inside the body. To this end, let an applied average strain tensor $\overline{\tilde{\varepsilon}_0(s)}$ be arbitrarily prescribed in the unit cell. Then, use the following average consistency condition (Mura, 1987) in the Laplace domain (i.e. the equivalence between the stress in the homogeneous material and the heterogeneous one):

$$\tilde{C}'(s) : (\overline{\tilde{\varepsilon}_0(s)} + P(s) : \tilde{C}(s) : \overline{\tilde{\varepsilon}^*(s)}) = \tilde{C}(s) : (\overline{\tilde{\varepsilon}_0(s)} + (P(s) : \tilde{C}(s) - I^{(4)}) : \overline{\tilde{\varepsilon}^*(s)}), \tag{6}$$

where $I^{(4)}$ is the identity fourth order tensor. It is worth noting that the tensor $P(s)$ takes into account the geometry of the inclusion and can be evaluated only once for every C' of the fibers. Then from eqn (6), the equivalent average volume eigenstrain $\overline{\tilde{\varepsilon}^*(s)}$ can be obtained in terms of the tensors $\tilde{C}'(s)$, $\tilde{C}(s)$, $P(s)$ and $\tilde{\varepsilon}_0$ for every s as:

$$\overline{\tilde{\varepsilon}^*(s)} = [((\tilde{C}(s) - \tilde{C}'(s))^{-1} - P(s)) : \tilde{C}(s)]^{-1} : \overline{\tilde{\varepsilon}_0(s)}. \tag{7}$$

The Laplace transform of the uniform overall stress $\sigma_0(s)$ in the unit cell is:

$$s\tilde{C}^*(s):\overline{\tilde{\varepsilon}_0(s)} = s\tilde{C}(s):(\overline{\tilde{\varepsilon}_0(s)} - V_f\overline{\tilde{\varepsilon}^*(s)}), \tag{8}$$

where $\tilde{C}^*(s)$ is the overall relaxation tensor of the composite material. Using eqn (7) and noting that $\overline{\varepsilon_0(s)}$ is arbitrary, the following expression of $\tilde{C}^*(s)$ is obtained:

$$s\tilde{C}^*(s) = s\tilde{C}(s) - sV_f((\tilde{C}(s) - \tilde{C}'(s))^{-1} - P(s))^{-1}. \tag{9}$$

Taking into account the periodicity of the microstructure and using a Fourier series representation of the field variables, the tensor $P(s)$ for an isotropic matrix can be written as [see Nemat-Nasser and Hori (1993) and Luciano and Barbero (1994a) for the elastic case]:

$$P(s) = \frac{1}{\tilde{\mu}_0(s)} \sum_{\xi}^{\pm\infty'} t(\xi)\left(\mathrm{sym}\,(\bar{\xi} \otimes I^{(2)} \otimes \bar{\xi}) - \frac{1}{2(1-v_0)}(\bar{\xi} \otimes \bar{\xi} \otimes \bar{\xi} \otimes \bar{\xi})\right), \tag{10}$$

where

$$t(\xi) = V_f\left(\frac{g_0(\xi)}{V_\Omega}\right)\left(\frac{g_0(-\xi)}{V_\Omega}\right) \quad \text{with} \quad g_0(\xi) = \int_\Omega \exp\,(i\xi x)\,\mathrm{d}x \tag{11}$$

and $\tilde{\mu}_0(s)$ and v_0 are the Laplace transform of the shear modulus and the Poisson ratio of the matrix respectively, $\xi = \{\xi_1, \xi_2, \xi_3\}$ with $\xi_j = 2\pi n_j/a_j (n_j = 0, \pm 1, \pm 2 \ldots j$ not summed, $j = 1, 2, 3)$ and $\bar{\xi} = \xi/|\xi|$. Then, from eqn (10), eqn (9) can be written as:

$$s\tilde{C}^*(s) = s\tilde{C}(s) - V_f\Bigg[(s\tilde{\lambda}_0(s)I^{(2)} \otimes I^{(2)} + s\tilde{\mu}_0(s)I^{(4)} - C')^{-1}$$
$$- \frac{1}{s\tilde{\mu}_0(s)} \sum_{\xi}^{\pm\infty'} t(\xi)\left((\mathrm{sym}(\bar{\xi} \otimes I^{(2)} \otimes \bar{\xi}) - \frac{1}{2(1-v_0)}(\bar{\xi} \otimes \bar{\xi} \otimes \bar{\xi} \otimes \bar{\xi}))\right)\Bigg]^{-1}, \tag{12}$$

where $\tilde{\lambda}_0(s)$ is the Laplace transform of the Lamé constant $\lambda(t)$ of the matrix. Then, defining the series S_l (with $l = 1$–9) as in Luciano and Barbero (1994a), the final expressions of the components of the tensor $\tilde{C}^*(s)$ can be obtained for any shape of inclusion. However, in this paper we are interested in composite materials reinforced with long fibers which may be transversely isotropic. In this case five series are different from zero and only three are independent (Nemat-Nasser *et al.*, 1982). If the fibers are aligned with the x_1-axis, it holds that:

$$S_1 = S_4 = S_8 = S_9 = 0,$$
$$S_2 = S_3, \quad S_5 = S_6. \tag{13}$$

Defining $\hat{\lambda}_0 = s\tilde{\lambda}_0(s)$ and $\hat{\mu}_0 = s\tilde{\mu}_0(s)$, the components of eqn (12) become:

$$s\tilde{C}^*_{11}(s) = \hat{\lambda}_0 + 2\hat{\mu}_0$$
$$- V_f(-a_4^2 + a_3^2)\left(-\frac{(2\hat{\mu}_0 + 2\hat{\lambda}_0 - C'_{33} - C'_{23})(a_4^2 - a_3^2)}{a_1} + \frac{2(a_4 - a_3)(\hat{\lambda}_0 - C'_{12})^2}{a_1^2}\right)^{-1}$$

$$s\tilde{C}^*_{12}(s) = \hat{\lambda}_0 + V_f\left(\frac{(\hat{\lambda}_0 - C'_{12})(a_4 - a_3)}{a_1}\right)$$
$$\times\left(\frac{(2\hat{\mu}_0 + 2\hat{\lambda}_0 - C'_{33} - C'_{23})(a_3^2 - a_4^2)}{a_1} + \frac{2(a_4 - a_3)(\hat{\lambda}_0 - C'_{12})^2}{a_1^2}\right)^{-1}$$

$$s\tilde{C}^*_{22}(s) = \hat{\lambda}_0+2\hat{\mu}_0 - V_{\mathrm{f}}\left(\frac{(2\hat{\mu}_0+2\hat{\lambda}_0-C'_{33}-C'_{23})a_3}{a_1} - \frac{(\hat{\lambda}_0-C'_{12})^2}{a_1^2}\right)$$

$$\times\left(\frac{(2\hat{\mu}_0+2\hat{\lambda}_0-C'_{33}-C'_{23})(a_3^2-a_4^2)}{a_1}+\frac{2(a_4-a_3)(\hat{\lambda}_0-C'_{12})^2}{a_1^2}\right)^{-1}$$

$$s\tilde{C}^*_{23}(s) = \hat{\lambda}_0 + V_{\mathrm{f}}\left(\frac{(2\hat{\mu}_0+2\hat{\lambda}_0-C'_{33}-C'_{23})a_4}{a_1} - \frac{(\hat{\lambda}_0-C'_{12})^2}{a_1^2}\right)$$

$$\times\left(\frac{(2\hat{\mu}_0+2\hat{\lambda}_0-C'_{33}-C'_{23})(a_3^2-a_4^2)}{a_1}+\frac{2(a_4-a_3)(\hat{\lambda}_0-C'_{12})^2}{a_1^2}\right)^{-1}$$

$$s\tilde{C}^*_{44}(s) = \hat{\mu}_0 - V_{\mathrm{f}}\left(\frac{2}{2\hat{\mu}_0-C'_{22}+C'_{23}} - \left(2S_3-\frac{4S_7}{2-2\nu_0}\right)\hat{\mu}_0^{-1}\right)^{-1}$$

$$s\tilde{C}^*_{66}(s) = \hat{\mu}_0 - V_{\mathrm{f}}\left((\hat{\mu}_0-C'_{66})^{-1}-\frac{S_3}{\hat{\mu}_0}\right)^{-1}, \tag{14}$$

where

$$a_1 = 4\hat{\mu}_0^2-2\hat{\mu}_0C'_{33}+6\hat{\lambda}_0\hat{\mu}_0-2C'_{11}\hat{\mu}_0-2\hat{\mu}_0C'_{23}+C'_{23}C'_{11}$$
$$+4\hat{\lambda}_0C'_{12}-2C'^2_{12}-\hat{\lambda}_0C'_{33}-2C'_{11}\hat{\lambda}_0+C'_{11}C'_{33}-\hat{\lambda}_0C'_{23}$$

$$a_2 = 8\hat{\mu}_0^3-8\hat{\mu}_0^2C'_{33}+12\hat{\mu}_0^2\hat{\lambda}_0-4\hat{\mu}_0^2C'_{11}-2\hat{\mu}_0C'^2_{23}+4\hat{\mu}_0\hat{\lambda}_0C'_{23}+4\hat{\mu}_0C'_{11}C'_{33}$$
$$-8\hat{\mu}_0\hat{\lambda}_0C'_{33}-4\hat{\mu}_0C'^2_{12}+2\hat{\mu}_0C'^2_{33}-4\hat{\mu}_0C'_{11}\hat{\lambda}_0+8\hat{\mu}_0\hat{\lambda}_0C'_{12}$$
$$+2\hat{\lambda}_0C'_{11}C'_{33}+4C'_{12}C'_{23}\hat{\lambda}_0-4C'_{12}C'_{33}\hat{\lambda}_0-2\hat{\lambda}_0C'_{11}C'_{23}$$
$$-2C'_{23}C'^2_{12}+C'^2_{23}C'_{11}+2C'_{33}C'^2_{12}-C'_{11}C'^2_{33}+\hat{\lambda}_0C'^2_{33}-\hat{\lambda}_0C'^2_{23}$$

$$a_3 = \frac{4\hat{\mu}_0^2+4\hat{\lambda}_0\hat{\mu}_0-2C'_{11}\hat{\mu}_0-2\hat{\mu}_0C'_{33}-C'_{11}\hat{\lambda}_0-\hat{\lambda}_0C'_{33}-C'^2_{12}+C'_{11}C'_{33}+2\hat{\lambda}_0C'_{12}}{a_2}$$
$$-\frac{S_3-\dfrac{S_6}{2-2\nu_0}}{\hat{\mu}_0}$$

$$a_4 = -\frac{-2\hat{\mu}_0C'_{23}+2\hat{\lambda}_0\hat{\mu}_0-\hat{\lambda}_0C'_{23}-C'_{11}\hat{\lambda}_0-C'^2_{12}+2\hat{\lambda}_0C'_{12}+C'_{11}C'_{23}}{a_2}+\frac{S_7}{\hat{\mu}_0(2-2\nu_0)}. \tag{15}$$

The series S_3, S_6, S_7 were computed by Nemat-Nasser *et al.* (1982) for several values of the volume fraction of the inclusions. The numerical values can be fitted with the following parabolic expressions using a least-square method (Luciano and Barbero, 1994a):

$$S_3 = 0.49247-0.47603V_{\mathrm{f}}-0.02748V_{\mathrm{f}}^2$$
$$S_6 = 0.36844-0.14944V_{\mathrm{f}}-0.27152V_{\mathrm{f}}^2$$
$$S_7 = 0.12346-0.32035V_{\mathrm{f}}+0.23517V_{\mathrm{f}}^2. \tag{16}$$

4. RELAXATION TENSOR IN THE TIME DOMAIN

The components of the relaxation tensor in the Laplace domain are given in eqn (14) for composite materials reinforced with transversely isotropic fibers. The matrix can be represented by any linearly viscoelastic material. The inversion to the time domain can be performed numerically (Bellman, 1966; Schapery, 1974; Laws and McLaughlin, 1978; Yancey and Pindera, 1990) or analytically if the matrix is represented with the two- or four-parameter model (Barbero and Luciano, 1994). Experimental data for viscoelastic matrix are usually available in the form of creep strain rate as a function of time t for various values of applied stress σ. A viscoelastic material can be represented by the power law

$$\dot{\varepsilon} = At^{B}\sigma^{D}. \tag{17}$$

The power law provides very good representation of the data, and the data reduction is very simple, with the coefficients A, B and D obtained simply by multiple linear regression. However, the power law in eqn (17) leads to nonlinear models for which the correspondence principle, used in this work, cannot be applied. Several simplified models can be derived from eqn (17). If the coefficient D is assumed to be equal to one, the material is said to be linearly viscoelastic. Such material is amenable to a well developed viscoelastic theory, based on the correspondence principle (Christensen, 1979). In the following, we use a linearly viscoelastic material represented by a power law of time, for which the creep compliance can be written

$$J(t) = D_0 + Ct^{n} \tag{18}$$

where D_0 represents the elastic response, and C and n are determined by linear regression of the logarithm of the creep data. Defining

$$\hat{J}(s) = s\tilde{J}(s) = D_0 + \frac{C\Gamma(n+1)}{s^{n}}, \tag{19}$$

then

$$\hat{E}(s) = s\tilde{E}(s) = \frac{1}{\hat{J}(s)}, \tag{20}$$

from which the Lamé functions of the matrix in the transformed domain are

$$\hat{\lambda}_0 = \frac{\hat{E}\nu_0}{(1+\nu_0)(1-2\nu_0)} \tag{21}$$

$$\hat{\mu}_0 = \frac{\hat{E}}{2(1+\nu_0)}. \tag{22}$$

Introducing these expressions in eqn (14), the coefficients of the relaxation tensor are obtained as functions of the Laplace variable s. The inversion to the time domain is done numerically (Bellman, 1966) using Legendre polynomials to find N discrete values of the solution. Then, a power law is fitted to these N discrete points using a chi-square goodness-of-fit statistical test. The initial value theorem is used to obtain the elastic response. For example, we consider a graphite–epoxy composite material containing transversely isotropic fibers (T300 graphite) represented by the following properties (Yancey and Pindera, 1990):

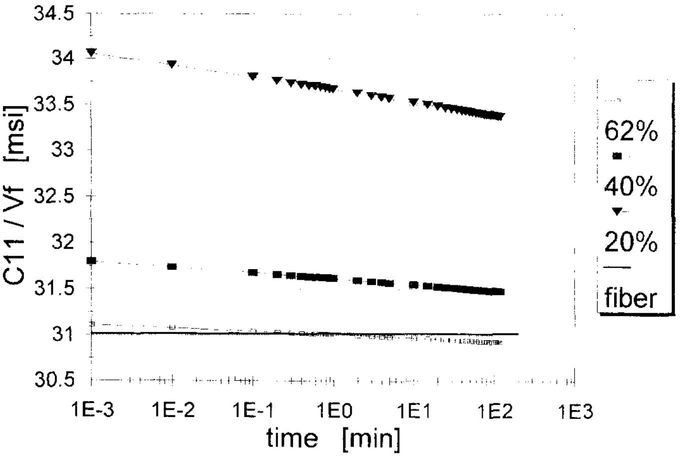

Fig. 2. Coefficient $C^*_{11}(t)$ of the relaxation tensor.

$$
\begin{aligned}
E_A &= 29.42 \times 10^6 \text{psi} \\
E_T &= 3.6 \times 10^6 \text{psi} \\
\nu_A &= 0.443 \\
\nu_T &= 0.05 \\
G &= 6.40 \times 10^6 \text{psi} \\
V_f &= 0.62
\end{aligned} \tag{23}
$$

and a viscoelastic matrix, represented by a power law with $D_0 = 1.53$ $1/(10^6\text{psi})$, $C = 0.093$ $1/(10^6\text{psi-min})$, $n = 0.17$, and $\nu_0 = 0.311$. The experimental data (Yancey and Pindera, 1990) are available for only 120 min. Plots of the components of the relaxation tensor are shown in Figs 2–7 for various values of fiber volume fraction. The component C^*_{11} in Fig. 2 is divided by the fiber volume fraction V_f to be able to show all results in a single figure. The values of C^f_{11} of the fiber are included for comparison. It can be noted that $C^*_{11}/V_f \simeq C^f_{11}$ when $V_f = 0.62$, but as V_f decreases, the contribution of the matrix becomes more important and the curves of C^*_{11}/V_f become separated from the fiber value C^f_{11}. The relaxation of the matrix is included in Fig. 3 for comparison. The effect of the reinforcement results in larger values of C^*_{22} for larger values of V_f. However, the relaxation rate with time is very similar to that of the matrix. Similar observations can be drawn for the remaining components of the relaxation tensor.

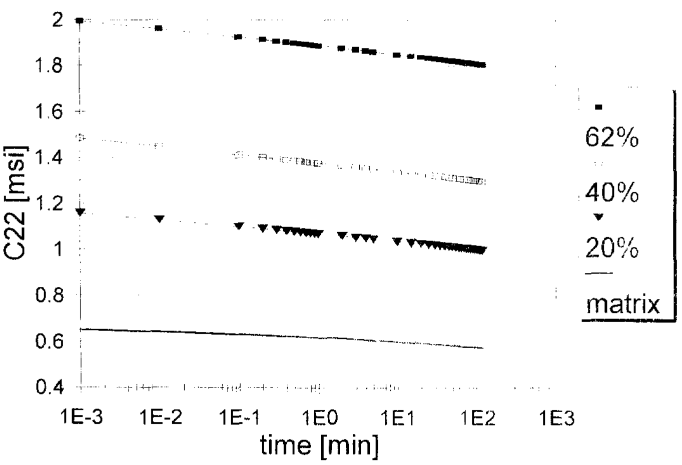

Fig. 3. Coefficient $C^*_{22}(t)$ of the relaxation tensor.

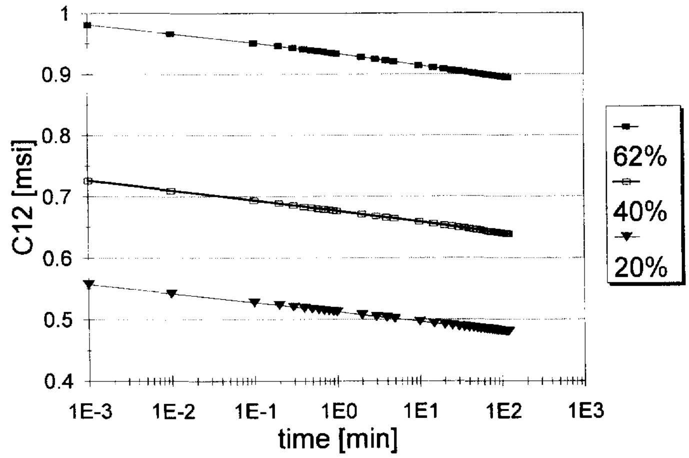

Fig. 4. Coefficient $C^*_{12}(t)$ of the relaxation tensor.

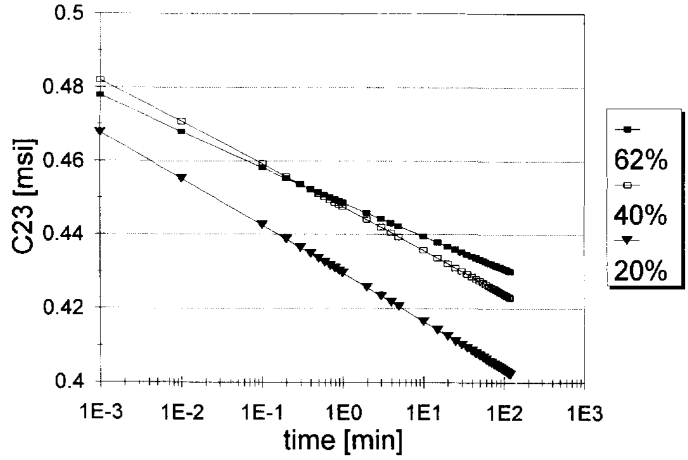

Fig. 5. Coefficient $C^*_{23}(t)$ of the relaxation tensor.

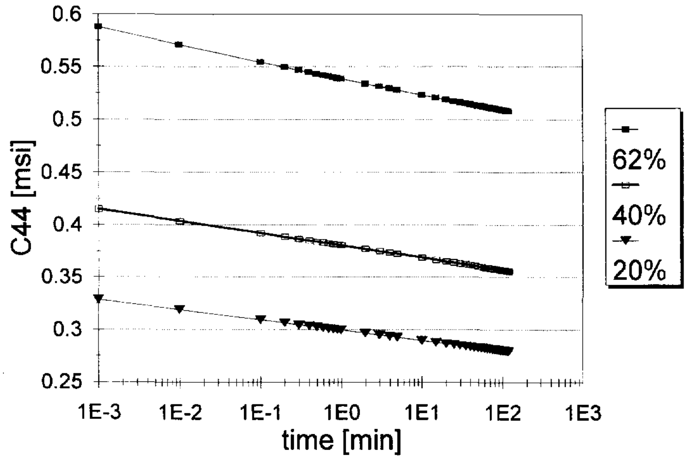

Fig. 6. Coefficient $C^*_{44}(t)$ of the relaxation tensor.

5. TRANSVERSELY ISOTROPIC COMPOSITE MATERIAL

Because of the periodicity of the microstructure, the relaxation tensor $C^*(t)$ for a unidirectional composite represents an orthotropic material with square symmetry. In the case considered in Section 3, the directions x_2 and x_3 are equivalent and the relaxation

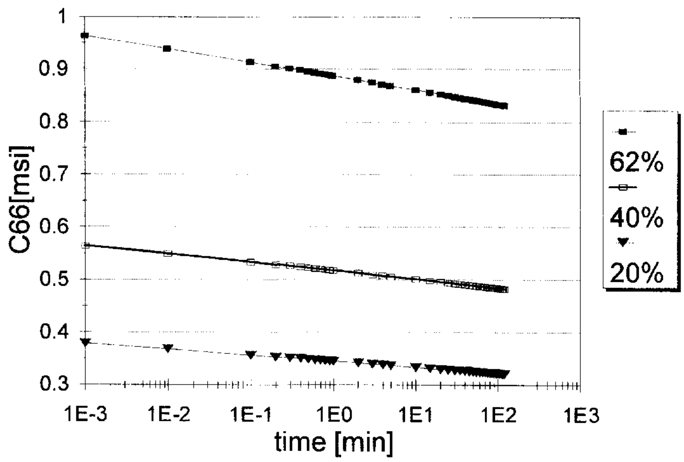

Fig. 7. Coefficient $C^*_{66}(t)$ of the relaxation tensor.

tensor is unchanged by a rotation about x_1 of $n\pi/2$ ($n = 0, \pm 1, \pm 2\ldots$). This implies that only six components are required to describe completely the tensor.

In order to obtain a transversely isotropic relaxation tensor $C^{*\mathrm{TI}}(t)$, equivalent in an average sense to the relaxation tensor with square symmetry, the averaging procedure proposed by Aboudi (1991) is used. Then, the following expressions are obtained explicitly in terms of the coefficients of the tensor $C^*(t)$ described in Section 3:

$$\begin{aligned}
C_{11}^{*\mathrm{TI}}(t) &= C^*_{11}(t)\\
C_{12}^{*\mathrm{TI}}(t) &= C^*_{12}(t)\\
C_{22}^{*\mathrm{TI}}(t) &= \tfrac{3}{4}C^*_{22}(t)+\tfrac{1}{4}C^*_{23}(t)+\tfrac{1}{2}C^*_{66}(t)\\
C_{23}^{*\mathrm{TI}}(t) &= \tfrac{1}{4}C^*_{22}(t)+\tfrac{3}{4}C^*_{23}(t)-\tfrac{1}{2}C^*_{66}(t)\\
C_{44}^{*\mathrm{TI}}(t) &= \tfrac{1}{2}(C^*_{22}(t)-C^*_{23}(t))\\
C_{66}^{*\mathrm{TI}}(t) &= C^*_{66}(t).
\end{aligned} \tag{24}$$

6. CREEP COMPLIANCE

The creep compliance in the Laplace domain can be obtained from the relaxation moduli [eqn (14)] by using:

$$S^*(s) = \frac{1}{s^2}C^*(s)^{-1}. \tag{25}$$

Plots of the components of the creep compliance tensor are shown in Figs 8–13. The compliance of the matrix is included in Fig. 9 for comparison. It can be noted that the reinforcement changes the creep behavior even at low values of the fiber volume fraction. The effect of the volume fraction is consistent among all the results presented in Figs 8–13.

7. COMPARISONS WITH EXPERIMENTAL RESULTS

Comparisons with experimental results are presented in this section for epoxy 934 reinforced with a 62% fiber volume fraction of T300 graphite fibers. Yancey and Pindera (1990) measured the viscoelastic properties of 934 epoxy resin at two temperatures [NR: room temperature (72°F); NE: elevated temperature (250°F)], as shown in Table 1.

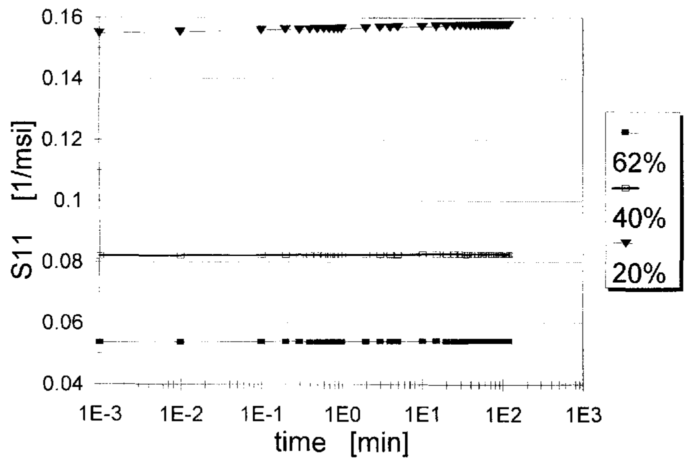

Fig. 8. Coefficient $S^*_{11}(t)$ of the creep tensor.

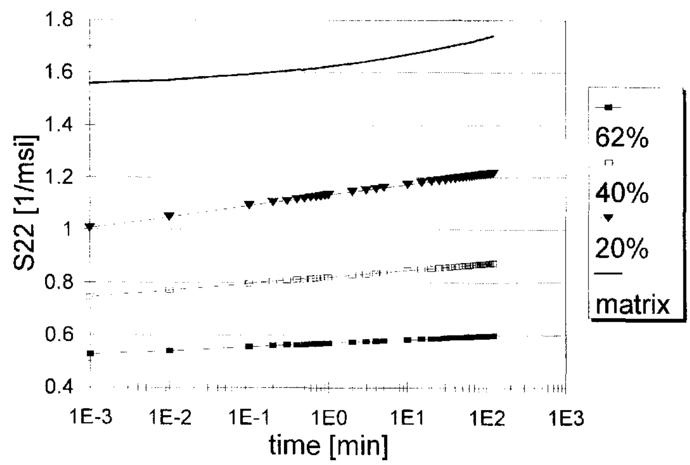

Fig. 9. Coefficient $S^*_{22}(t)$ of the creep tensor.

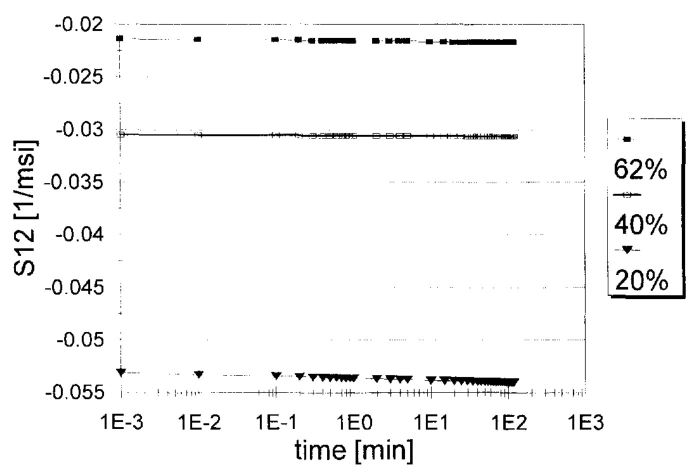

Fig. 10. Coefficient $-S^*_{12}(t)$ of the creep tensor.

The fiber is T300 graphite with properties back calculated from composite properties so that the micromechanic predictions of the method of cells (Aboudi, 1991) used by Yancey and Pindera (1990) fit the experimental data. Minor discrepancies between the experimental results and the model results presented in this work can be attributed to the fact that the fiber properties were back calculated to fit exactly Aboudi's model (1991) and

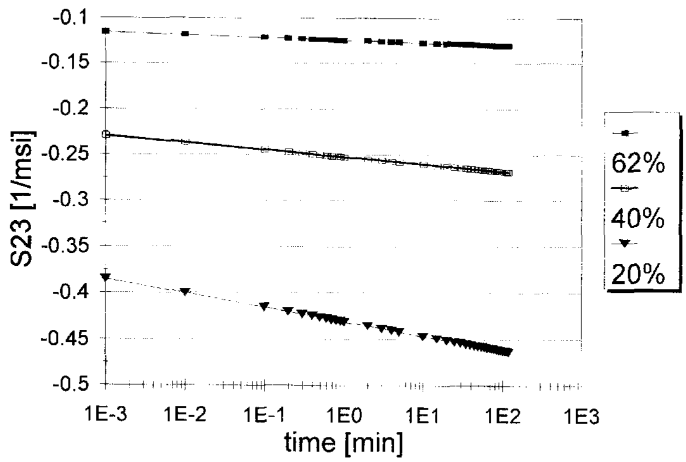

Fig. 11. Coefficient $-S^*_{23}(t)$ of the creep tensor.

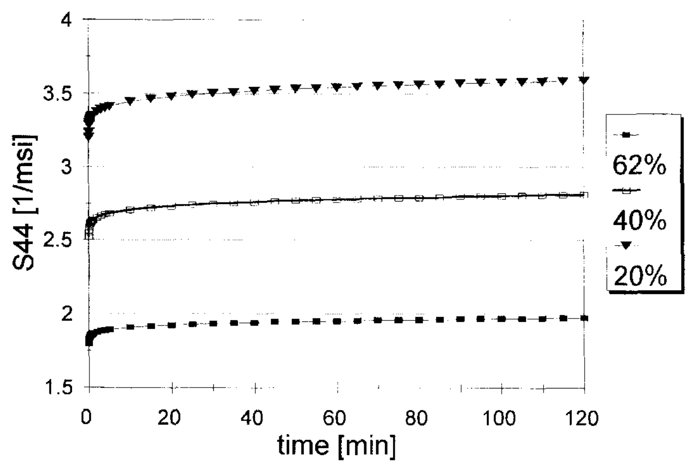

Fig. 12. Coefficient $S^*_{44}(t)$ of the creep tensor.

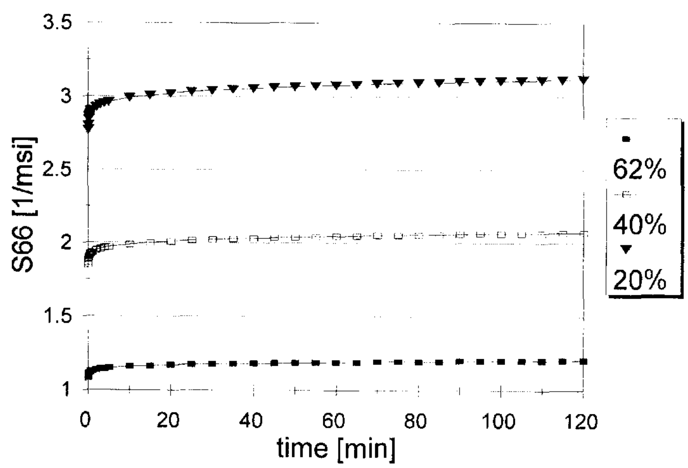

Fig. 13. Coefficient $S^*_{66}(t)$ of the creep tensor.

not the initial response of the periodic microstructure model presented in this work. The fiber properties (Yancey and Pindera, 1990) are shown in Table 2.

The predictions obtained with the present model, using the properties given in Tables 1 and 2, are compared with the experimental data in Figs 14–17. The experimental data, available only up to 120 min, were obtained from tests performed on coupons with fiber

Table 1

	NR	NE
$D_0(1/10^6\text{psi})$	1.530	2.05
$C\,(1/10^6\text{psi}-\text{min})$	0.093	0.173
n	0.170	0.200
ν_0	0.311	0.317

Table 2

	NR	NE
$E_A(10^6\text{psi})$	29.42	31.09
$E_T(10^6\text{psi})$	3.67	2.15
ν_A	0.443	0.450
ν_T	0.05	0.05
$G\,(10^6\text{psi})$	6.40	9.89

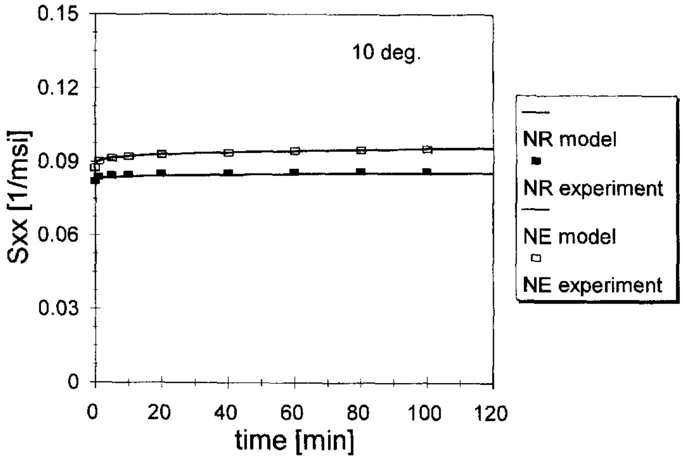

Fig. 14. Comparison with experimental results of axial creep response in the global system of axes (10°).

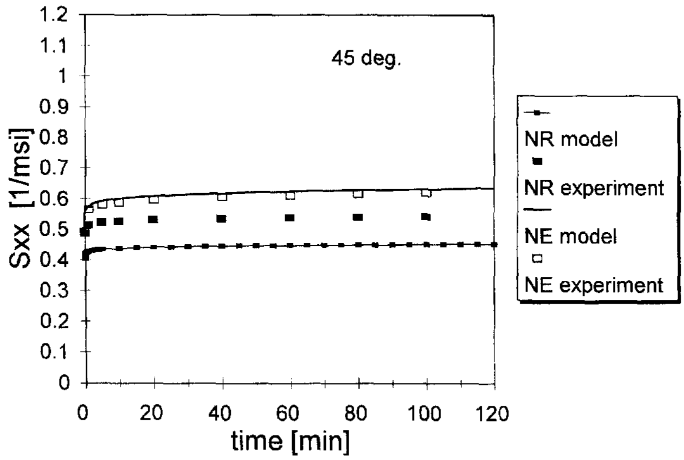

Fig. 15. Comparison with experimental results of axial creep response in the global system of axes (45°).

orientations of 10°, 45° and 90°, as indicated in Figs 14–17. Since experimental results for S_{66} from tests at 10° and 45° are almost coincident, all the data are collapsed in Fig. 16. While the compliance of the composite at 10° from the fiber direction S_{xx} is fiber dominated,

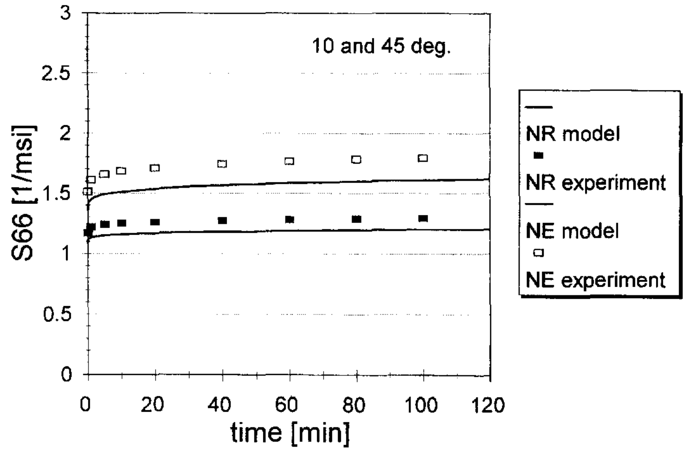

Fig. 16. Comparison with experimental results of shear creep response in the material system of axes (10° and 45°).

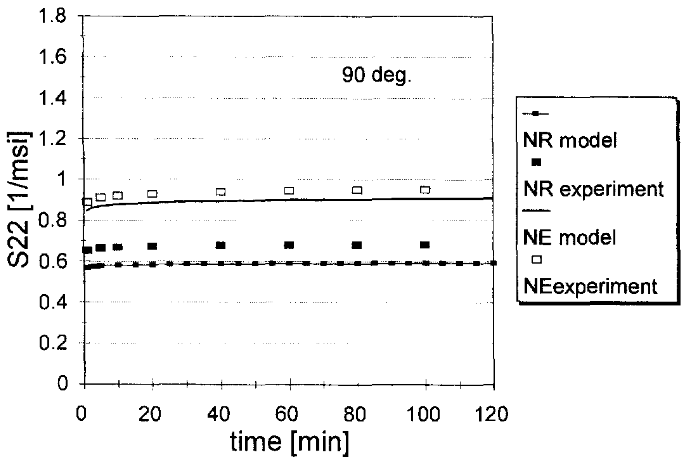

Fig. 17. Comparison with experimental results of transverse creep response in the material system of axes (90°).

the compliances S_{66} (inverse of the in plane shear modulus) and S_{22} are matrix dominated properties (Figs 16 and 17). The comparison presented demonstrates the ability of the proposed model to represent the viscoelastic behavior of fiber-reinforced composite materials.

8. CONCLUSIONS

Analytical expressions for the Laplace transform of the relaxation and creep tensors of composite materials reinforced with transversely isotropic fibers and linear viscoelastic matrix are presented. Numerical inversion to the time domain was implemented along with a power law representation of the matrix creep. Good agreement with available experimental data is obtained. The interaction effects between the constituents and the geometry of the inclusions are fully accounted for by the micromechanical model used.

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